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Non-isothermal steady-state diffusion within Glaser's condensation model

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Abstract

Non-isothermal diffusion models for water vapor transfer through building materials have been examined and their results compared with isothermal models. The Glaser condensation scheme with incorporated non-isothermal diffusion has been presented and its issues studied.

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1. Introduction

Although the Glaser model [1–7] was introduced in the second half of the 1950s as the combined graphical and numerical method to assess the condensation of water vapor in building structures, it is still used in building practice [8,9]. Its aim is to estimate the amount of condensate gathered during winter and the amount of water vapor evaporated during summer. If the condensate does not exceed specified limits [8,9] and its amount is lower than that of evaporated water, the building structure is considered as convenient.

However, it is well known that Glaser's model suffers from some drawbacks which make the model rather debatable. For example, the model includes neither hygroscopic nor liquid transports and omits transition from liquid into solid phase as well. In addition, it is based on isothermal diffusion. But the real building envelopes, especially in winter season, are considerable non-isothermal $T(x)$. Under other circumstances, when the *saturated* partial pressure curve is constructed, the model respects the non-isothermal conditions. These inconsistencies (isothermal versus non-isothermal conditions) rise the question: what will happen with the model if the fully non-isothermal conditions are incorporated into its scheme.

The goal of the present contribution is to implement the non-isothermal diffusion on the standard Glaser's model and discuss some features of the modified scheme.

2. Non-isothermal diffusion of water vapor in building porous materials

Let us suppose that the building envelope through which the diffusion flux goes is represented by a plain brick wall of the thickness $d = 44$ cm. The wall separates a heated room of a usual environment (*surface* temperature and relative humidity: $T_1 = 293.15$ K, $\varphi_1 = 60\%$ RH) from an outdoor space ($T_2 = 255.15$ K $\varphi_2 = 84\%$ RH). The atmospheric pressure of *dry* air is usually considered to be approximately the same at both sides ($p_a = 98066.5$ Pa used in [10]). The atmospheric pressure p consists of partial pressures of water vapor p_w and dry air p_a

$$p = p_w + p_a, \quad p \ll p_a, \ p \approx p_a. \tag{1}$$

Similar relations hold for *mass concentrations* as well

$$c = c_w + c_a, \quad c_w \ll c_a, \ c \approx c_a. \tag{2}$$

Reasonable choice [11] for the temperature profile $T(x)$ of a wall is a linear function

$$T(x) = T_1 \frac{T_1 - T_2}{d} x \quad (T_1 > T_2). \tag{3}$$

The first Fick's law for non-isothermal diffusion assumes the following form [12]:

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$$\vec{q}_{\mathrm{w}} = y_{\mathrm{w}}(\vec{q}_{\mathrm{w}} + \vec{q}_{\mathrm{a}}) - cD_{\mathrm{wa}}(T)\nabla y_{\mathrm{w}},\quad y_{\mathrm{w}} = \frac{c_{\mathrm{w}}}{c} = \frac{c_{\mathrm{w}}}{c_{\mathrm{w}} + c_{\mathrm{a}}}, \tag{4}$$

where $\vec{q}_{\mathrm{w}}$ and $\vec{q}_{\mathrm{a}}$ are diffusion fluxes of water vapor and air, respectively, and $D_{\mathrm{wa}}(T)$ is the temperature-dependent diffusivity. The diffusion flow should fulfil the condition of continuous flow

$$c\frac{\partial y_{\mathrm{w}}}{\partial t} = \nabla\vec{q} \tag{5}$$

which is the second Fick law. Assuming an unidirectional steady-state diffusion ($\partial y_{\mathrm{w}}/\partial t = 0$) along the x-axis and a negligibly small diffusion flow ($q_{\mathrm{a}} \to 0$) of heavy air molecules ($N_2 + O_2 + \cdots$) as compared with lighter H_2O molecules ($q_{\mathrm{w}} \gg q_{\mathrm{a}}$), the two Fick equations can be rewritten as follows:

$$q_{\mathrm{w}} = -\frac{cD_{\mathrm{wa}}(T)}{1 - y_{\mathrm{w}}(1+r)}\frac{\mathrm{d}y_{\mathrm{w}}}{\mathrm{d}x},\quad \frac{q_{\mathrm{a}}}{q_{\mathrm{w}}} = r \ll 1, \tag{6}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}q_{\mathrm{w}} = 0 \Rightarrow q_{\mathrm{w}} = \mathrm{const.} \tag{7}$$

with the boundary conditions

$$y_{\mathrm{w}}(0) = y_{1\mathrm{w}},\quad y_{\mathrm{w}}(d) = y_{2\mathrm{w}}. \tag{8}$$

Following the work of Schirmer [10] and Krischer and co-worker [13] the diffusivity of water vapor in air pores can be expressed as the function of temperature

$$D = \frac{k}{\mu}T^{n},\quad n = 1.81,\quad k = 8.9718 \times 10^{-10}\ \mathrm{m}^2\,\mathrm{s}^{-1}\,\mathrm{K}^{-1.81} \tag{9}$$

where μ is the 'diffusion resistance factor'—a purely material constant corresponding to the wall material. According to the gas law the mass concentration c is a function of pressure and temperature $\mathrm{J\,kg^{-1}\,K^{-1}}$

$$c = \frac{p}{R_{\mathrm{wa}}T} \approx \frac{p_{\mathrm{a}}}{R_{\mathrm{a}}T},\quad R_{\mathrm{a}} = \frac{R}{M_{\mathrm{a}}} \approx 297\ \mathrm{J\,kg^{-1}\,K^{-1}},\quad R = 8.31\ \mathrm{J\,mol^{-1}\,K^{-1}}. \tag{10}$$

From Eqs. (3)–(10) it follows:

$$\frac{p_{\mathrm{a}}k}{R_{\mathrm{a}}\mu}\int_{y_{1\mathrm{w}}}^{y_{\mathrm{w}}(x)}\frac{\mathrm{d}y_{\mathrm{w}}}{1 - y_{1\mathrm{w}}(1+r)} = -q_{\mathrm{w}}\int_0^x\left[T_1 + \frac{T_1 - T_2}{\mathrm{d}}x\right]^{1-n}\mathrm{d}x. \tag{11}$$

Inserting the second boundary condition (8) into (11) one obtains the diffusion flux

$$q_{\mathrm{w}} = \frac{p_{\mathrm{a}}k(2-n)}{R_{\mathrm{a}}d\mu(1+r)}\cdot\frac{T_1 - T_2}{T_1^{2-n} - T_2^{2-n}}\cdot\ln\left[\frac{1 - y_{2\mathrm{w}}(1+r)}{1 - y_{1\mathrm{w}}(1+r)}\right],\quad n = 1.81 \tag{12}$$

which—together with (11)—gives the vapor profile $y_{\mathrm{w}}(x)$

$$y_{\mathrm{w}}(x) = \frac{1}{1+r}\left\{1 - [1 - y_{1\mathrm{w}}(1+r)]\cdot\left[\frac{1 - y_{2\mathrm{w}}(1+r)}{1 - y_{1\mathrm{w}}(1+r)}\right]^{(T_1^{2-n} - ((T_1 - ((T_1 - T_2)/d)x)^{2-n})/(T_1^{2-n} - T_2^{2-n})}\right\},\quad n = 1.81. \tag{13}$$

Taking into account the *diffusion through an immobilized air layer* (*DIAL* model), i.e. $q_{\mathrm{a}} \to 0$ ($r \to 0$), relations (12), (13) can be simplified

$$q_{\mathrm{w}}^{*} \approx \frac{0.19p_{\mathrm{a}}k}{R_{\mathrm{a}}d\mu}\frac{T_1 - T_2}{T_1^{0.19} - T_2^{0.19}}\cdot\ln\left[\frac{1 - y_{2w}}{1 - y_{1w}}\right], \tag{14}$$

$$y_{\mathrm{w}}^{*}(x) \approx \left\{1 - (1 - y_{1w})\cdot\left[\frac{1 - y_{2w}}{1 - y_{1w}}\right]^{(T_1^{0.19} - (T_1 - (T_1 - T_2)/d)x)^{0.19}/(T_1^{0.19} - T_2^{0.19})}\right\}. \tag{15}$$

Relation (14) may be formally rewritten as follows:

$$q_{\mathrm{w}}^{*} \approx \frac{\ln\left[\frac{1 - y_{2\mathrm{w}}}{1 - y_{1\mathrm{w}}}\right]}{R_{\mathrm{eff}}^{*}},\quad R_{\mathrm{eff}}^{*} = \frac{d}{D_{\mathrm{eff}}^{*}}\ (\mathrm{kg^{-1}\,m^2\,s}), \tag{16}$$

where

$$D_{\mathrm{eff}}^{*} = \frac{0.19kp_{\mathrm{a}}}{\mu R_{\mathrm{a}}}\cdot\frac{T_1 - T_2}{T_1^{2-n} - T_2^{2-n}} = \frac{5.629 \times 10^{-8}}{\mu}\cdot\frac{T_1 - T_2}{T_1^{0.19} - T_2^{0.19}}\ (\mathrm{kg\,m^{-1}\,s^{-1}}).$$

Since $y_{1\mathrm{w}} \ll 1$ and $y_{2\mathrm{w}} \ll 1$ one can further simplified

$$\ln\left[\frac{1 - y_{2\mathrm{w}}}{1 - y_{1\mathrm{w}}}\right] \approx y_{1\mathrm{w}} - y_{2\mathrm{w}},\quad q_{\mathrm{w}}^{*} \approx \frac{y_{1\mathrm{w}} - y_{2\mathrm{w}}}{R_{\mathrm{eff}}^{*}}\ (\mathrm{kg\,m^{-2}\,s^{-1}}). \tag{17}$$

3. DRAL model

Relations (14)–(17) hold within the framework of the *DIAL* approximation when the air layer embedded in a porous material is only slightly perturbed by the diffusion of water vapor molecules that possess smaller mass and much lower concentration. It seems to be natural that not only the pressure of dry air remains constant but also the concentrations of dry and wet air vary only slightly across the wall. This can be nicely illustrated when the concentration ratios are calculated for a particular case, e.g. for the internal and external conditions defined at the beginning of the previous section

$$\begin{aligned}&c_{\mathrm{w}}^{\mathrm{inner}}/c_{\mathrm{w}}^{\mathrm{outer}} = 16.23,\\&c_{\mathrm{a}}^{\mathrm{inner}}/c_{\mathrm{a}}^{\mathrm{outer}} = 0.870,\\&c^{\mathrm{inner}}/c^{\mathrm{outer}} = 0.883.\end{aligned} \tag{18}$$

From (18) it can be seen that the variations of air concentrations c_{a}, c across the wall are negligibly small and, thus, the profile $c_{\mathrm{a}}(x), c(x)$ may be considered as approximately horizontal, i.e. constant for common climatic conditions. This does not hold at all for water vapor whose concentration vary considerably and, thus, its profile shows clear functional dependence on x

$$c_{\mathrm{w}} = f(x), \quad c_{\mathrm{a}} \approx \text{const.}, \quad c \approx \text{const.}, \quad c_{\mathrm{w}}(x) \ll c_{\mathrm{a}}. \tag{19}$$

If these relations are included into Fick's equation (6), one can obtain a more simplified transport equation

$$q_{\mathrm{d}} = -D_{\mathrm{wa}}(T)\frac{\mathrm{d}c_{\mathrm{w}}(T,p_{\mathrm{w}})}{\mathrm{d}x}, \quad 1-y_{\mathrm{w}} \approx 1, \quad r \approx 0, \quad c \approx \text{const.} \tag{20}$$

An analogous transport equation holds for non-isothermal diffusion of a gas in a solid *compact* body, therefore, the present approximation might be termed as *d*iffusion through a '*r*igid' *a*ir *l*ayer (*DRAL* model). At first sight this approximation might seem rather unrealistic but the final account of all physical factors and their *approximate behavior* lead to such conclusion. Eq. (20) is quite analogous to those presented in technical literature for the cases when the total concentration is constant (see e.g. Eq. 16.2-3 in [12]). The assumption $c = \text{const.}$ does not necessarily mean the condition for isothermal state. Nearly constant total concentration c can be expected not only with non-isothermal diffusion of a gas in a solid *compact* body which does not contain any air pores but also in solid materials containing *closed* pores (cavities) filled with air. Diffusion flux in such materials goes both through the airless solid structures whose concentration (density) is almost unaffected by temperature and through voids with a constant air content (constant concentration) provided the walls of voids are hardly penetrable for heavy air molecules in contrast to lighter water ones. The building foam materials like foam polyethylene approach this type of material. It seems to be probable that the DRAL model might be more applicable just to such materials. However, it is necessary to stress that if a strong non-isothermal state causes essential variations in the total concentration profile, i.e. $c = f(x)$, the DRAL model will fail to determine a realistic diffusion flux. Shortly speaking, the transport equation (20) should enable to approximate the diffusion flux of water vapor in various materials only under the usual climatic conditions since such conditions represent a weakly non-isothermal state which does not disturb essentially the total concentration profile. Nevertheless, under highly non-isothermal state, the DRAL model can yield correct results only with quite special materials. Numerical comparison of the results obtained by various models is presented in Section 5.

Let us continue determining the solution of differential equation (20). First it is necessary to specify the complete set of the Fick equations within the DRAL approximation

$$q_{\mathrm{d}} = -\frac{D}{R_{\mathrm{w}}}\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{p_{\mathrm{w}}(x)}{T(x)}\right), \quad c_{\mathrm{w}} = \frac{p_{\mathrm{w}}}{R_{\mathrm{w}}T}, \quad R_{\mathrm{w}} = 462\ \mathrm{J\,kg^{-1}\,K^{-1}}, \tag{21}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{k}{\mu R_{\mathrm{w}}}T^{n}(x)\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{p_{\mathrm{w}}(x)}{T(x)}\right)\right] = 0, \quad n = 1.81 \tag{22}$$

and the boundary conditions belonging to a non-isothermal wall of the thickness d

$$\frac{p_{\mathrm{w}}(0)}{T(0)} = \frac{p_{1\mathrm{w}}}{T_1}, \quad \frac{p_{\mathrm{w}}(d)}{T(d)} = \frac{p_{2\mathrm{w}}}{T_2}. \tag{23}$$

Since the non-linear temperature profile $T(x)$ appears in common walls only under special conditions [11], linear profile is assumed here as well

$$T(x) = T_1 - \frac{T_1 - T_2}{d}x = a - bx. \tag{24}$$

Inserting (24) into Fick's equations (21), (22) and taking into account the first boundary condition (23), the corresponding solution can be found

$$\frac{k}{\mu R_{\mathrm{w}}}(a-bx)^{n}\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{p_{\mathrm{w}}}{T}\right) = -q_{\mathrm{d}} = \text{const.}, \tag{25}$$

$$\int_{p_{1\mathrm{w}}/T_1}^{p_{\mathrm{w}}(x)/T(x)} \mathrm{d}\left(\frac{p_{\mathrm{w}}}{T}\right) = \int_0^x -\frac{q_{\mathrm{d}}R_{\mathrm{w}}\mu}{k}(a-bx)^{-n}\,\mathrm{d}x, \tag{26}$$

$$q_{\mathrm{d}} = \frac{\left[\frac{p_{1\mathrm{w}}}{T_1} - \frac{p_{\mathrm{w}}(x)}{T(x)}\right]kb(1-n)}{\mu R_{\mathrm{w}}\left[a^{1-n} - (a-bx)^{1-n}\right]}. \tag{27}$$

Inserting the second boundary condition (23) into (27), we can express the steady-state diffusion flux q_{d}

$$q_{\mathrm{d}} = \frac{\left(\frac{p_{1\mathrm{w}}}{T_1} - \frac{p_{2\mathrm{w}}}{T_2}\right)kb(1-n)}{\mu R_{\mathrm{w}}\left[a^{1-n} - (a-bd)^{1-n}\right]} \tag{28}$$

which goes through a non-isothermal wall with the linear temperature profile (24). The symbols a and b in (28) can be specified using (24)

$$g_{\mathrm{d}} = \frac{k(1-n)}{\mu R_{\mathrm{w}} d} \frac{\left(\frac{p_{1\mathrm{w}}}{T_1} - \frac{p_{2\mathrm{w}}}{T_2}\right)(T_1 - T_2)}{T_1^{1-n} - T_2^{1-n}}, \quad n = 1.81. \tag{29}$$

Relation (29) can be rearranged and the effective diffusion resistance R_{d} and effective coefficient D_{eff} may be introduced

$$\begin{aligned} q_{\mathrm{d}} &= \frac{c_{1\mathrm{w}} - c_{2\mathrm{w}}}{R_{\mathrm{d}}} \ (\mathrm{kg\,m^{-2}\,s^{-1}}), \\ R_{\mathrm{d}} &= \frac{d}{D_{\mathrm{eff}}} \ (\mathrm{m^{-1}\,s}), \\ D_{\mathrm{eff}} &= \frac{k(n-1)(T_1 - T_2)}{\mu(T_2^{1-n} - T_1^{1-n})} \ (\mathrm{m^2\,s^{-1}}), \end{aligned} \tag{30}$$

where

$$c_{1\mathrm{w}} = \frac{p_{1\mathrm{w}}}{R_{\mathrm{w}} T_1} \ (\mathrm{kg\,m^{-3}}), \quad c_2 = \frac{p_{2\mathrm{w}}}{R_{\mathrm{w}} T_2} \ (\mathrm{kg\,m^{-3}}).$$

By means of relations (30) the non-isothermal diffusion flux q_{d} expressed within DRAL approximation can be easily calculated.

For completeness, the partial pressure profile $p_{\mathrm{w}}(x)$ inside the wall should be presented. Functions $p_{\mathrm{w}}(x)$ and $c_{\mathrm{w}}(x)$ are given by Eqs. (26) and (28)

$$p_{\mathrm{w}}(x) = \left(T_1 - \frac{T_1 - T_2}{d}x\right)\left[\frac{p_{1\mathrm{w}}}{T_1} - \left(\frac{p_{1\mathrm{w}}}{T_1} - \frac{p_{2\mathrm{w}}}{T_2}\right) \times \frac{T_1^{1-n} - \left(T_1 - \frac{T_1 - T_2}{d}x\right)^{1-n}}{T_1^{1-n} - T_2^{1-n}}\right], \tag{31}$$

$$c_{\mathrm{w}}(x) = \frac{p_{1\mathrm{w}}}{T_1} - \left(\frac{p_{1\mathrm{w}}}{T_1} - \frac{p_{2w}}{T_2}\right)\frac{T_1^{1-n} - \left(T_1 - \frac{T_1 - T_2}{d}x\right)^{1-n}}{T_1^{1-n} - T_2^{1-n}} \tag{32}$$

for $n = 1.81$ the profiles $p_{\mathrm{w}}(x)$ and $c_{\mathrm{w}}(x)$ will read

$$p_{\mathrm{w}}(x) = \left(T_1 - \frac{T_1 - T_2}{d}x\right)\left[\frac{p_{1\mathrm{w}}}{T_1} - \left(\frac{p_{1\mathrm{w}}}{T_1} - \frac{p_{2\mathrm{w}}}{T_2}\right) \times \frac{T_1^{-0.81} - \left(T_1 - \frac{T_1 - T_2}{d}x\right)^{-0.81}}{T_1^{-0.81} - T_2^{-0.81}}\right], \tag{33}$$

$$c_{\mathrm{w}}(x) = c_{1\mathrm{w}} - (c_{1\mathrm{w}} - c_{2\mathrm{w}})\frac{T_1^{-0.81} - \left(T_1 - \frac{T_1 - T_2}{d}x\right)^{-0.81}}{T_1^{-0.81} - T_2^{-0.81}}. \tag{34}$$

At first sight it is obvious that the $p_{\mathrm{w}}(x)$ and $c_{\mathrm{w}}(x)$ profiles are not linear. Nevertheless, for usual temperature and partial pressure differences between outdoor and indoor spaces in the Central European climatic region the graphs of $p_{\mathrm{w}}(x)$ and $c_{\mathrm{w}}(x)$ will closely follow the linear behavior as it can be easily verified.

4. IM-TDR and IM-TIR models

The Glaser standard condensation model [7] is based on isothermal diffusion, i.e. the temperature of a wall is considered to be a constant T_{m} and equal to the mean value of surface temperatures

$$T_{\mathrm{m}} = \frac{T_1 + T_2}{2}. \tag{35}$$

The Fick's equations for diffusion in an isothermal structure can be obtained from (21) and (22) after inserting $T = T_{\mathrm{m}}$

$$q_{\mathrm{d}} = -\frac{D}{R_{\mathrm{w}} T_m}\frac{\mathrm{d}p_{\mathrm{w}}}{\mathrm{d}x}, \frac{\mathrm{d}q_{\mathrm{d}}}{\mathrm{d}x} = 0, \tag{36}$$

where

$$D = \frac{k}{\mu}T_{\mathrm{m}}^n, \quad n = 1.81.$$

Using the following boundary conditions:

$$p_{\mathrm{w}}(0) = p_{1\mathrm{w}}, \quad p_{\mathrm{w}}(d) = p_{2\mathrm{w}} \tag{37}$$

the solution of (36) can be found easily

$$q_{\mathrm{d}} = \frac{p_{1\mathrm{w}} - p_{2\mathrm{w}}}{R_{\mathrm{d}}}, \quad R_{\mathrm{d}} = \frac{\mu d}{\delta}, \ \delta = \frac{k T_{\mathrm{m}}^{n-1}}{R_{\mathrm{w}}}, \ n = 1.81, \tag{38}$$

$$p(x) = p_{1\mathrm{w}} - \frac{p_{1\mathrm{w}} - p_{2\mathrm{w}}}{d}x. \tag{39}$$

As it can be seen, the pressure profile $p(x)$ is a linear function of x in contrast to non-isothermal profiles (15) and (34) that are non-linear. In spite of the fact that Fick's equation (36) hold exactly for isothermal structure only, i.e. for constant temperature, the thermal resistance R_{d} is actually temperature dependent $R_{\mathrm{d}}(T_{\mathrm{m}})$. Such a model may be termed as *isothermal model with temperature dependent resistance*, i.e. IM-TDR.

Nevertheless, in civil engineering practice the isothermal Fick equations are combined with diffusion resistance that is not temperature dependent because the average temperature is fixed to the one-year average, i.e. $T_{\mathrm{m}} = 283.15$ K, regardless of the actual value T_{m} of a particular structure. Such a model may be called the *isothermal model with temperature independent resistance*, i.e. *IM-TIR*.

5. The region of numerical compatibility

To analyse behavior of all the models discussed, their results for the structure given at the beginning of Section 2 are presented in Fig. 1 and Table 1. The calculations were done under the assumption that the external partial pressure of water vapor remained constant while the external temperature was successively increasing, i.e.

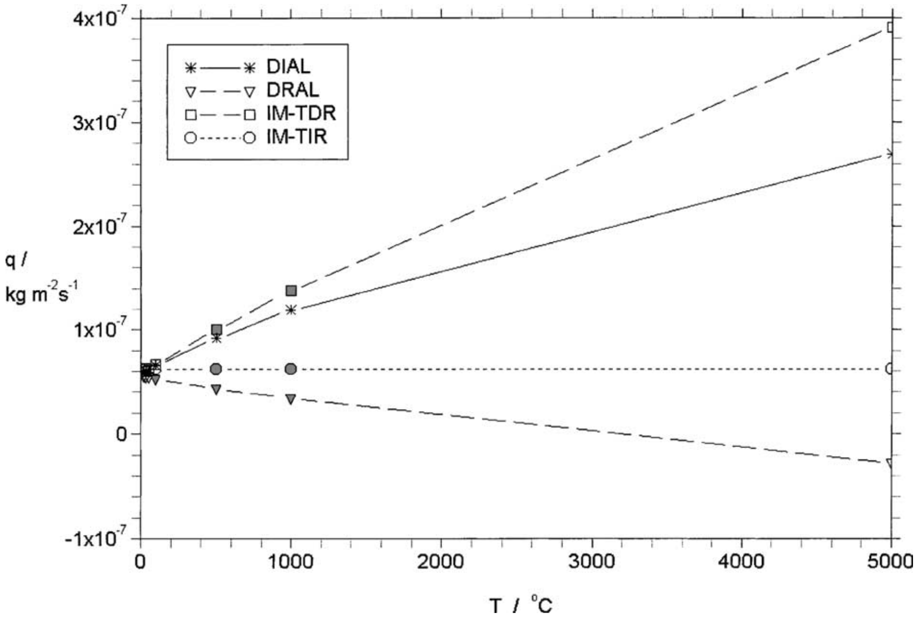

Fig. 1. Diffusion fluxes in dependence on temperatures.

Table 1
Diffusion fluxes through the brick wall ($\mu = 9$) separating external space ($\varphi = 84\%$R.H., $T_2 = 255.18$ K, $p_{2w} = 105$ Pa) and internal room with increasing temperature (20–5000 °C)

Quantity	293.15 K (20 °C)	303.15 K (30 °C)	323.15 K (50 °C)	373.15 K (100 °C)	773.15 K (500 °C)	1273.15 K (1000 °C)	5273.15 K (5000 °C)
p_w (Pa)	1402.2	1402.2	1402.2	1402.2	1402.2	1402.2	1402.2
$c_w \times 10^3$ (kg m^{-3})	10.350	10.000	9.392	8.1340	3.9300	2.3800	5.7600
$y_w \times 10^3$	9.1	9.1	9.1	9.1	9.1	9.1	9.1
IM-TDR, $R_d \times 10^{-9}$ (m s^{-1})	21.61	21.61	21.29	19.35	12.98	9.42	3.32
DIAL, $R^*_{eff} \times 10^{-4}$ (kg m^{-1} s^{-1})	14.18	13.98	13.61	12.8	9.13	7.07	3.13
DRAL, $R_{eff} \times 10^{-4}$ (m^2 s^{-1})	17.13	16.62	15.68	13.75	7.00	4.38	1.15
IM-TIR, $q \times 10^8$ (kg m^{-2} s^{-1})	6.19	6.19	6.19	6.19	6.19	6.19	6.19
IM-TDR, $q \times 10^8$ (kg m^{-2} s^{-1})	6.00	6.00	6.10	6.70	10.00	13.77	39.07
DIAL, $q \times 10^8$ (kg m^{-2} s^{-1})	5.93	6.02	6.18	6.57	9.21	11.9	26.87
DRAL, $q \times 10^8$ (kg m^{-2} s^{-1})	5.52	5.48	5.42	5.27	4.34	3.40	−2.83

external relative humidity was decreasing. Such conditions enable to study the influence of temperature difference $\Delta T = T_1 - T_2$ on the numerical compatibility of the models.

From both Table 1 and Fig. 1 it can be seen that at 'small' temperature differences ($\Delta T < 40$ K) the heat fluxes of all four models differ by several percent only and, thus, under such 'quasi-isothermal' conditions all the models give nearly identical results. Since the temperature difference ΔT in the Central European climatic region does not usually exceed 50 K during a year, all the models seem to be applicable within the civil engineering practice.

Nevertheless, if strong non-isothermal conditions are established ($\Delta T \gg 40$ K), it is necessary to carefully distinguish between computational models. While the IM-TIR scheme is not applicable at all under such conditions, the applicability for the remaining three models will depend on material structure. For materials with macroscopic open pores filled with air (like silicate building materials or mineral wool) the most convenient models seem to be DIAL and IM-TDR, the first of which should be given priority to in practical calculations. The applicability of the DRAL model under strong non-isothermal conditions is more problematic since it requires a constant total concentration profile to be established which is fulfilled only in very special cases.

6. Glaser's condensation scheme

The Glaser graphical method [7] for finding condensation region inside a building structure enables to incorporate straightforwardly not only isothermal diffusion (coordinate system (R_d, p))—as it was initially introduced by Glaser—but also non-isothermal diffusion like DIAL (coordinate system (R^*_{eff}, y_w)) and DRAL (coordinate system (R_{eff}, c_w)). Table 2 and Figs. 2–4 show the results of Glaser's scheme applied within the isothermal and non-isothermal diffusions to the building structure specified in Section 2. Since the structure is only 'weakly' non-isothermal ($\Delta T < 40$ K), one cannot expect large differences in results.

The amount of condensate Δq_{AB} extracted from the area of 1 m^2 per one second is calculated as the difference between the diffusion flux q_A entering the condensation region in point A (see Figs. 2–4) and leaving the region (q_B) in point B

$$\Delta q_{AB} = q_A - q_B. \tag{40}$$

The corresponding amounts of condensate Δq_{AB} determined by the three investigated models (standard Glaser's model: 4.21×10^{-8} kg m^{-2} s^{-1}; DIAL: 4.4×10^{-8} kg m^{-2} s^{-1}; DRAL: 3.5×10^{-8} kg m^{-2} s^{-1}) show slightly

Table 2
Condensation region according to Glaser's method (building structure specified in Section 2)

x (m)	T (K)	Glaser's standard model			DIAL model			DRAL model		
		$R_d \times 10^{-9}$ (m s^{-1})	p_w (Pa)	$p_w^{(\text{satur.})}$ (Pa)	$R^*_{\text{eff}} \times 10^{-4}$ (kg m^{-1} s^{-1})	$y_w \times 10^3$	$y_w^{(\text{satur.})} \times 10^3$	$R_{\text{eff}} \times 10^{-4}$ (m^{-1} s)	$c_w \times 10^3$ (kg m^{-3})	$c_w^{(\text{satur.})} \times 10^3$ (kg m^{-3})
0	293.15	0	1402	2337	0	9.108	15.09	0	10.35	17.25
0.0733	286.82	3.60	1186	1564	2.255	7.770	10.15	2.568	8.930	11.80
0.1467	280.48	7.20	969.8	1025	4.554	6.405	6.674	5.245	7.452	7.910
0.2200	274.15	10.81	753.6	657	6.892	5.018	4.288	8.029	5.915	5.187
0.2933	267.81	14.41	537.4	390	9.278	3.600	2.524	10.937	4.310	3.150
0.3667	261.48	18.01	321.2	224	11.706	2.156	1.466	13.967	2.637	1.850
0.4400	255.15	21.61	105	125	14.183	0.688	0.8187	17.133	0.895	1.060
		$\Delta q_{AB} = 4.21 \times 10^{-8}$ (kg m^{-2} s^{-1}) (from Fig. 2)			$\Delta q_{AB} = 4.4 \times 10^{-8}$ (kg m^{-2} s^{-1}) (from Fig. 3)			$\Delta q_{AB} = 3.5 \times 10^{-8}$ (kg m^{-2} s^{-1}) (from Fig. 4)		

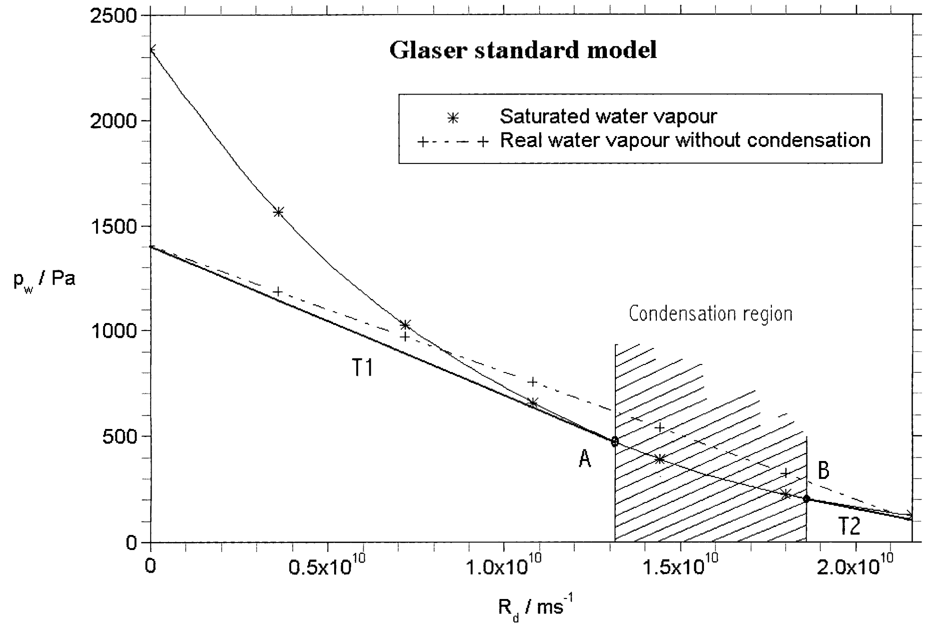

Fig. 2. Glaser's standard condensation scheme.

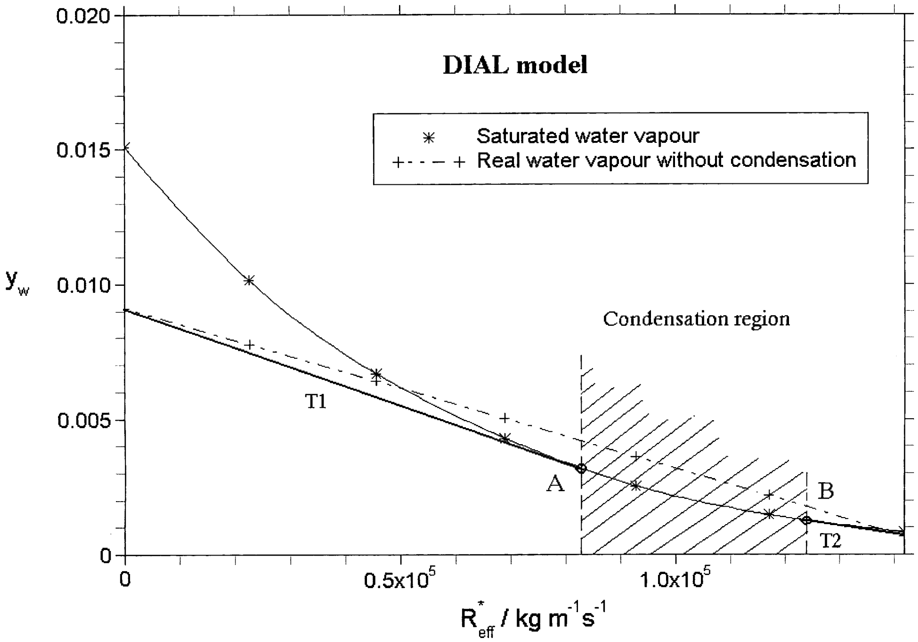

Fig. 3. Glaser's condensation scheme with DIAL model.

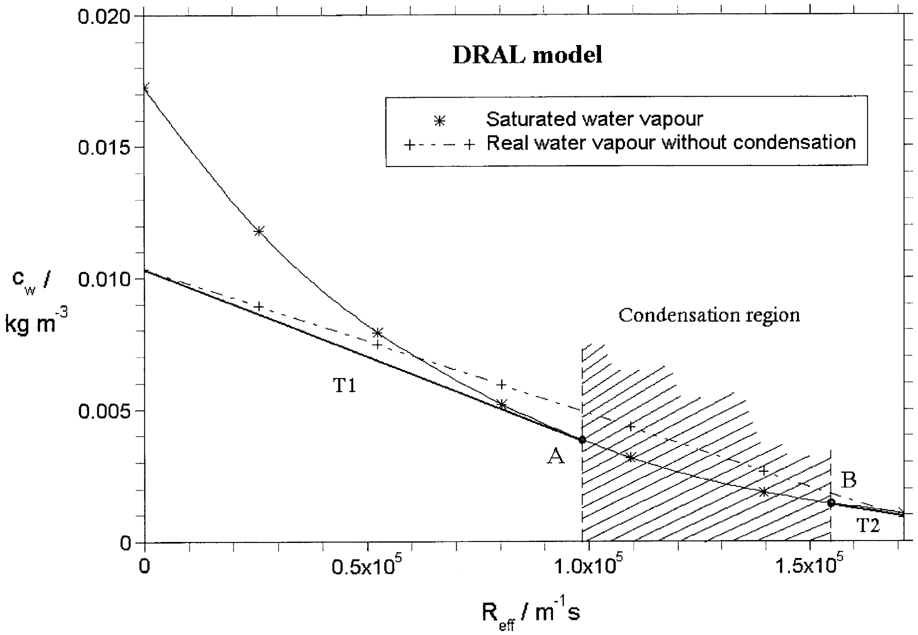

Fig. 4. Glaser's condensation scheme with DRAL model.

larger differences but still laying under 20% which is acceptable accuracy in this field. Essential differences should be expected under strong non-isothermal conditions. However, with silicate porous materials the highest numerical reliability should be ascribed to the condensation scheme with the DIAL model incorporated.

7. Conclusion

It has been illustrated that non-isothermal calculations of condensate and diffusion fluxes of water vapor in building structures being under common Central European climatic conditions ($\Delta T < 40$) do not lead to quite different results as compared with isothermal calculations. This is because of rather small temperature gradient established across the structures. Nevertheless, in special cases when the structure is exposed to very large temperature differences ($\Delta T > 40$ K) it is necessary to chose a convenient non-isothermal model to determine diffusion flux of water vapour. For porous silicate structures one of these convenient models may be represented by the DIAL approximation.

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